

# Homodyne detection for measuring coherent phase state distributions

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## Abstract

Using coherent phase states, parameterized phase state distributions for a single-mode radiation field are introduced and their integral relation to the phase-parameterized field-strength distributions is studied. The integral kernel is evaluated and the problem of direct sampling of the coherent phase state distributions using balanced homodyne detection is considered. Numerical simulations show that when the value of the smoothing parameter is not too small the coherent phase state distributions can be obtained with sufficiently well accuracy. With decreasing value of the smoothing parameter the determination of the coherent phase state distributions may be an effort, because both the numerical calculation of the sampling function and the measurement of the field-strength distributions are required to be performed with drastically increasing accuracy.

# 1 Introduction

The quantum-mechanical description of amplitude and phase quantities and their measurement has turned out to be troublesome and is still of matter of discussion (for a review, see Ref. [1]). Attempts to introduce amplitude and phase operators in quantum mechanics and particularly in the theory of the quantized radiation field are nearly as old as quantum theory. Since Dirac's introduction of amplitude and phase operators in 1927 [2], a series of concepts have been developed. Roughly speaking, there are two routes to introduce amplitude and phase quantities and the associated operators. In the first, amplitude and phase variables are introduced by intrinsic phase operator definitions [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14] or definitions closely related to phase space functions [15, 16, 17, 18, 19, 20, 21]. In the second, they are introduced by defining them more pragmatically from the output observed in phase-sensitive measurements [22, 23].

Phase measurements are usually based on some kinds of interference experiments. To our knowledge the earliest measurements of phase were carried out by Gerhardt, Büchler, and Litfin [24]. A powerful method pioneered by Walker and Carroll [25] for phase measurements has been homodyne and heterodyne detection techniques. These techniques, which were also used by Noh, Fougères, and Mandel [23] for defining measurement-assisted phase quantities, have offered qualitatively new possibilities. In particular, Smithey, Beck, Faridani, and Raymer [26] have demonstrated experimentally that balanced homodyne detection can be used to reconstruct the quantum state of a signal-field mode (see also Ref. [28]). The method is frequently called optical homodyne tomography. The Wigner function of the signal mode is reconstructed from the measured set of phase-parameterized field-strength distributions [27] using inverse Radon transformation techniques, and the density matrix of the signal field is then obtained by Fourier transforming the Wigner function. It should be noted that tomographic methods have also been applied to the reconstruction of quantum states of molecular vibrations by Dunn, Walmsley, and Mukamel [29]. Since the Wigner function or the density matrix contains all knowable information on the quantum statistics of the signal mode, they can be used for inferring phase distributions that have been not directly available from experiments, as was demonstrated by Beck, Smithey, and Raymer [30] (see also Ref. [31]).

The tomographic method for reconstructing the Wigner function and the density matrix of the signal mode from the phase-parameterized field-strength distributions measured in balanced homodyne detection (for the detection scheme see Refs. [32, 33, 34]) is based on the fact that knowledge of the field-strength distributions for all phases within a  $\pi$ -interval is equivalent to knowledge of the quantum state [35]. In other words, the measured set of field-strength distributions themselves can be regarded as being representative of the quantum state of the signal mode and the question arises

of whether or not quantum-phase distributions can be sampled directly from the measured field-strength distributions, without making the detour via the density matrix. The problem is similar to those related to the determination of the density matrix itself. Whereas in the first experiments the density matrix was obtained via the reconstructed Wigner function, recently methods of direct sampling of the density matrix from the measured data have been suggested and successfully applied [36, 37, 38, 39, 40, 41, 42, 43, 44, 45]. The feasibility of measuring quantities by direct sampling depends on the relationships between the quantities and the field-strength distributions recorded. In particular, these relationships should be stable against sampling errors, so that measurement of a sufficiently large set of data should enable one to obtain the quantities with sufficiently well precision. Clearly, different concepts of introduction of phase variables lead to different phase distributions and to different relations between them and the field-strength distributions. In the present paper we restrict attention to the class of quantum-phase distributions that can be defined on the basis of the coherent phase states [10, 13, 14]. Roughly speaking, these distributions are smoothed London phase state distributions which can be parameterized with respect to the degree of smoothing. The connection of the coherent phase state (CPS) distributions with the phase-parameterized field-strength distributions is considered and the problem of direct sampling of the CPS distributions from the field-strength distributions is studied. Numerical simulations are carried out to illustrate the applicability of the method.

In Sec. 2 the CPS distributions are introduced. Their integral relation to the phase-parameterized field-strength distributions is studied in Sec. 3. Results of a computer simulation of direct sampling of CPS distributions are reported in Sec. 4, and a summary and some concluding remarks are given in Sec. 5.

## 2 Coherent phase state distributions

In the long history of the quantum phase problem the eigenstates of the “exponential-phase” operator

$$\hat{E}_- = \sum_{n=0}^{\infty} |n\rangle \langle n+1| \quad (1)$$

( $|n\rangle$ , photon-number states) have been played a central role. In particular, the states

$$|e^{i\phi}\rangle = \sum_{n=0}^{\infty} e^{in\phi} |n\rangle \quad (2)$$

satisfying the eigenvalue equation

$$\hat{E}_- |e^{i\phi}\rangle = e^{i\phi} |e^{i\phi}\rangle \quad (3)$$

have been referred absolutely to as phase states. These states introduced by London [3] resolve the unity, but they are non-orthogonal and non-normalizable. As was shown by Lèvy-Leblond [10], the eigenvalue equation

$$\hat{E}_- |z\rangle = z |z\rangle \quad (4)$$

has also other solutions than those in Eq. (2), viz.

$$|z\rangle = \sqrt{1 - |z|^2} \sum_{n=0}^{\infty} z^n |n\rangle, \quad |z| < 1. \quad (5)$$

These normalizable states, which form an over-complete basis, have been re-examined and called coherent phase states by Shapiro and Shepard [13]. Their properties have also been studied by Vourdas [14] who showed that they are SU(1,1) Perelomov coherent states  $|z; k\rangle$  corresponding to the  $k = \frac{1}{2}$  representation [46]. In the limit  $|z| \rightarrow 1$  the states  $|z\rangle$  become the non-normalizable London states in Eq. (2):

$$|e^{i\phi}\rangle = \lim_{\epsilon \rightarrow 0} \frac{1}{\sqrt{1 - e^{-2\epsilon}}} |\phi, \epsilon\rangle, \quad (6)$$

where the notations

$$|\phi, \epsilon\rangle \equiv |z\rangle, \quad z = e^{-\epsilon} e^{i\phi} \quad (\epsilon > 0) \quad (7)$$

have been introduced.

The states  $|\phi, \epsilon\rangle$  can be used to define  $\epsilon$ -parameterized phase state distributions – CPS distributions – of a radiation-field mode via the overlap of its quantum state with coherent phase states:

$$p(\phi, \epsilon) = N^{-1}(\epsilon) \langle \phi, \epsilon | \hat{\rho} | \phi, \epsilon \rangle, \quad (8)$$

where

$$N(\epsilon) = \int_{-\pi}^{\pi} d\phi \langle \phi, \epsilon | \hat{\rho} | \phi, \epsilon \rangle \quad (9)$$

( $\hat{\rho}$ , density matrix of a radiation-field mode). In the limit  $\epsilon \rightarrow 0$  the distributions  $p(\phi, \epsilon)$  become the familiar London phase state distribution  $p(\phi, 0) = \langle e^{i\phi} | \hat{\rho} | e^{i\phi} \rangle$ . The CPS distributions  $p(\phi, \epsilon)$  can be regarded as smoothed London phase state distributions, which can be directly sampled from the field-strength distributions measured in balanced homodyne detection, as we will see below.

To illustrate the effect of the smoothing parameter  $\epsilon$ , in Figs. 1 – 3 CPS distributions and the London phase state distributions of coherent and squeezed states are plotted for various values of the coherent amplitude and the squeezing parameter. When the value of the smoothing parameter is decreased the CPS distributions become closer and closer to the corresponding London phase state distribution, which is observed, at least in tendency, in

the limit  $\epsilon \rightarrow 0$ . From comparison of Figs. 2 and 3 we further see that with increasing (mean) photon number the value of the smoothing parameter is required to be decreased in order to observe CPS distributions that are comparably close to the London phase state distribution. This can be explained as follows. In the expansion in photon-number states of the London phase states the weight of the states with large photon numbers is increased with increasing (mean) number of photons in the state under study. In the CPS distributions the overlap of these states with the state under study would be suppressed when the smoothing parameter would be unchanged.

### 3 Relation to the field-strength distributions

The phase-parameterized field-strength distributions measurable in perfect balanced homodyning,

$$p(\mathcal{F}, \varphi) = \langle \mathcal{F}, \varphi | \hat{\varrho} | \mathcal{F}, \varphi \rangle, \quad (10)$$

were introduced by Schubert and Vogel [27] (for details see, e.g., [37]). In Eq. (10), the  $|\mathcal{F}, \varphi\rangle$  are the eigenvectors of a field-strength operator  $\hat{F}(\varphi) = F\hat{a} + F^*\hat{a}^\dagger$  at chosen phase  $\varphi$  ( $F = |F|e^{-i\varphi}$ ,  $\hat{a}^\dagger$  and  $\hat{a}$  are the photon creation and destruction operators, respectively). It should be noted that non-perfect detection introduces additional noise, so that the measured distributions are, in general, smeared field-strength distributions. These are convolutions of the true field-strength distributions with Gaussians [34]:

$$p(\mathcal{F}, \varphi; s) = \int d\mathcal{F} p(\mathcal{F}', \varphi) p(\mathcal{F} - \mathcal{F}'; s), \quad (11)$$

where

$$p(\mathcal{F}; s) = \left(2\pi|s||F|^2\right)^{-\frac{1}{2}} \exp\left(-\frac{\mathcal{F}^2}{2|s||F|^2}\right) \quad (12)$$

and  $s = 1 - \eta^{-1}$ ,  $\eta$  being the detection efficiency ( $\eta \leq 1$ ).

The relation between the field-strength distributions and the CPS distributions may be obtained as follows. Using the expansion of the density operator  $\hat{\varrho}$  in  $s$ -parameterized displacement operators [47] and relating the averages of the  $s$ -parameterized displacement operators to the characteristic functions of the field-strength distributions [35] yields (see, e.g., Refs. [41, 42])

$$\hat{\varrho} = \int_0^\pi d\varphi \int d\mathcal{F} p(\mathcal{F}, \varphi; s) \hat{K}(\mathcal{F}, \varphi; -s), \quad (13)$$

where the operator integral kernel  $\hat{K}(\mathcal{F}, \varphi; -s)$  reads as

$$\hat{K}(\mathcal{F}, \varphi; -s) = \frac{|F|^2}{\pi} \int dy |y| \exp\left\{iy \left[\hat{F}(\varphi) - \mathcal{F}\right] - \frac{1}{2}sy^2|F|^2\right\}. \quad (14)$$

Note that Eq. (14) can formally be applied also to cases where the detection efficiency is less than unity. Combining Eqs. (8) and (13), we find that  $p(\phi, \epsilon)$  can be related to  $p(\mathcal{F}, \varphi)$  as

$$p(\phi, \epsilon) = N^{-1}(\epsilon) \int_0^\pi d\varphi \int_{-\infty}^\infty d\mathcal{F} p(\mathcal{F}, \varphi; s) K_\epsilon(\phi, \mathcal{F}, \varphi; s), \quad (15)$$

where

$$K_\epsilon(\phi, \mathcal{F}, \varphi; s) = \langle \phi, \epsilon | \hat{K}(\mathcal{F}, \varphi; -s) | \phi, \epsilon \rangle. \quad (16)$$

Equation (15) can be regarded as the basis equation for direct sampling of CPS distributions from the difference-count statistics in balanced homodyning. Let us consider the sampling function  $K_\epsilon(\phi, \mathcal{F}, \varphi; s)$  in more detail. Expanding the  $|\phi, \epsilon\rangle$  in photon-number states [see Eq. (5)], Eq. (16) can be rewritten as

$$K_\epsilon(\phi, \mathcal{F}, \varphi; s) = (1 - e^{-2\epsilon}) \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} K_{nm}(\mathcal{F}, \varphi; -s) e^{i(n-m)\phi} e^{-\epsilon(n+m)}, \quad (17)$$

where

$$K_{nm}(\mathcal{F}, \varphi; -s) = \langle n | \hat{K}(\mathcal{F}, \varphi; -s) | m \rangle \quad (18)$$

is the sampling function for determining the density matrix in the photon-number basis, which has been studied in detail in a number of papers [38, 39, 40, 41, 42, 43, 44]. It is given by

$$K_{nm}(\mathcal{F}, \varphi; -s) = e^{i(n-m)\varphi} f_{nm}(x; s), \quad (19)$$

$x = \mathcal{F}/(\sqrt{2}|F|)$ , where  $f_{nm}(x; s)$  may be expressed in terms of parabolic cylinder functions. From Eqs. (17) and (19) we see that  $K_\epsilon(\phi, \mathcal{F}, \varphi; s)$  is a  $2\pi$ -periodic function of the sum phase  $\phi + \varphi$ . Since the relation  $f_{nm}(x; s) = f_{mn}(x; s)$  is valid,  $K_\epsilon(\phi, \mathcal{F}, \varphi; s)$  is an even function of the sum phase. For  $\eta > 1/2$  the function  $f_{nm}(x, s)$  is bounded. For large values of  $x$  it behaves like  $x^{-|n-m|-2}$ , so that  $K_\epsilon(\phi, \mathcal{F}, \varphi; s)$  is bounded for  $\epsilon > 0$ . In Figs. 4(a) – 4(d)  $K_\epsilon(\phi, \mathcal{F}, \varphi; s)$  is shown for various values of the smoothing parameter  $\epsilon$  and the detection efficiency  $\eta$ .

Let us first restrict attention to perfect detection ( $\eta = 1$ , i.e.,  $s = 0$ ), Figs. 4(a) – 4(c). We see that with increasing  $\epsilon$  the fine structure in the dependence on  $\mathcal{F}$  of  $K_\epsilon(\phi, \mathcal{F}, \varphi; s)$  is lost. The fine structure obviously results from highly oscillating terms in Eq. (17), i.e., from terms with  $n, m \gg 1$ . These terms become more and more suppressed when  $\epsilon$  is increased. On the other hand, the width of the interval of  $\mathcal{F}$  in which  $K_\epsilon(\phi, \mathcal{F}, \varphi; s)$  is essentially nonzero slowly decreases with increasing  $\epsilon$ . Apart from finenesses, the CPS distributions may therefore be expected to reflect typical properties of the London phase state distribution even for values of  $\epsilon$  that are not extremely small. Clearly, for  $\epsilon \rightarrow 0$  the calculation of  $K_\epsilon(\phi, \mathcal{F}, \varphi; s)$  becomes an effort, because the number of terms in Eq. (17) drastically increases. From Fig. 4(d)

we see that in the case of non-perfect detection ( $\eta < 1$ , i.e.,  $s < 0$ ) the function  $K_\epsilon(\phi, \mathcal{F}, \varphi; s)$  becomes highly structurized, so that increasing effort has to be made to sample CPS distributions from the field-strength distributions.

In general, the CPS distributions  $p(\phi, \epsilon)$  are determined by the field-strength distributions  $p(\mathcal{F}, \varphi)$  in more or less extended intervals of  $\mathcal{F}$  and  $\varphi$ . In particular, Figs. 4(a) – (c) clearly show that the intervals of  $\mathcal{F}$  and  $\varphi + \phi$  in which  $K_\epsilon(\phi, \mathcal{F}, \varphi; s)$  is essentially nonzero are not concentrated in the vicinity of the points  $\mathcal{F} = 0$  and  $\varphi + \phi = \pi/2$ , respectively. Hence, the CPS distributions (and the London phase state distribution) cannot be obtained from the field-strength distributions at  $\mathcal{F} = 0$  and  $\varphi + \phi = \pi/2$ . This contradicts the operational phase distribution  $p(\phi) \propto p(\mathcal{F} = 0, \varphi = \pi/2 - \phi)$  introduced by Vogel and Schleich [18].

## 4 Direct sampling of coherent phase state distributions

As already mentioned, Eq. (15) can be used for direct sampling of CPS distributions from the field-strength distributions measured in balanced homodyne detection. To give an example, let us consider the determination of CPS distributions for a squeezed vacuum state. The result of a computer simulation of measurement of a set of field-strength distributions for a squeezed vacuum state with mean photon number  $\langle \hat{n} \rangle = 1$  is shown in Fig. 5. In the simulation perfect detection is considered and  $30 \times 10^4$  events at 30 phase values are assumed to be recorded. The feasibility of direct sampling of CPS distributions  $p(\phi, \epsilon)$  from the measured field-strength distributions is demonstrated in Fig. 6 for  $\epsilon = 0.1$ .

Comparing the sampled CPS distribution with the calculated distribution, we see that the measured double-peak structure is in good agreement with the theoretical prediction. In particular, in the intervals of phase in which reduced phase fluctuations are observed a sufficiently well agreement between theory and experiment is found. Relative large discrepancies between the measured and the calculated curves are seen in the intervals of phase in which enhanced phase fluctuations are observed. Clearly a better agreement and a higher accuracy can be achieved when the number of phases at which the field-strength distributions are measured and the number of events recorded at each phase are increased.

## 5 Summary and concluding remarks

We have studied the integral relation of the CPS distributions  $p(\phi, \epsilon)$  to the phase-parameterized field-strength  $p(\mathcal{F}, \varphi)$  measurable in balanced homodyne detection and calculated the corresponding integral kernel. Since

the CPS distributions can be regarded as smoothed London phase state distributions, the degree of smoothing being determined by the parameter  $\epsilon$  in the coherent phase states  $|z = e^{-\epsilon} e^{i\phi}\rangle$  ( $\epsilon > 0$ ), in the limit  $\epsilon \rightarrow 0$  the London phase state distribution is observed.

With decreasing value of the smoothing parameter  $\epsilon$  the information about the phase statistics is increased, because the smoothing parameter  $\epsilon$  controls the interval of the field strength used for “probing” the phase statistics of the state under consideration. With decreasing value of  $\epsilon$  this interval is increased and comprises the whole field-strength axis in the limit  $\epsilon \rightarrow 0$ . Hence, with decreasing value of  $\epsilon$  the determination of the integral kernel requires inclusion in the calculation of increasing values of the field strength. This point must be considered very carefully when the state under consideration contains large field-strength values that are desired to be included in an analysis of the phase statistics.

In the paper we have calculated the integral kernel using an expansion in the photon-number basis. Since in this basis with decreasing value of  $\epsilon$  the values of the photon numbers which must be taken into account increase, highly oscillating functions are involved in the expansion and much effort must be made to obtain results with reasonable accuracy. This difficulty might be overcome by using consequently a field-strength basis and avoiding the detour via the photon-number basis, as has recently been demonstrated for direct sampling of the density matrix in a field-strength basis [45].

The integral relation between the CPS distributions and the field-strength distributions can be used for direct sampling of the CPS distributions from the data measured in balanced homodyning, the (bounded) integral kernel playing the role of the sampling function. To illustrate the method, we have performed a computer simulation of measurement of a set of field-strength distributions and sampling from them the CPS distributions for a squeezed vacuum state with mean photon number  $\langle \hat{n} \rangle = 1$ . In this case a smoothing parameter of about 0.1 is sufficient to detect the features typical for the phase distribution of the state. Using realistic number of phases at which the field-strength distribution is measured and realistic numbers of events recorded the sampled CPS distributions are found to be in good agreement with the calculated ones.

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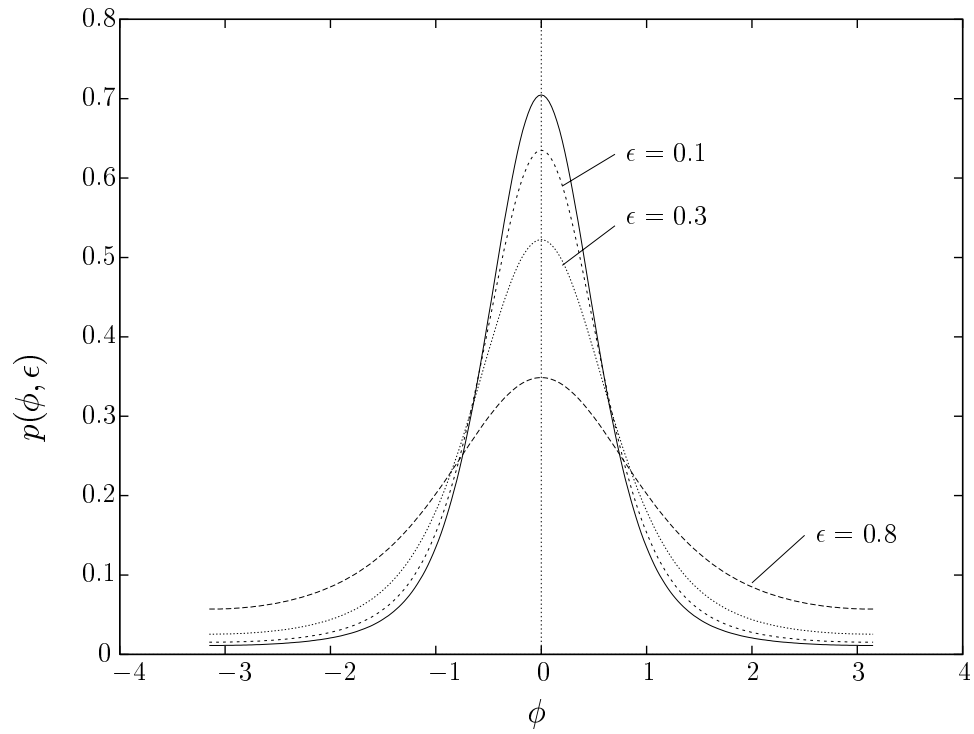


Figure 1: The CPS distributions  $p(\phi, \epsilon)$  for a coherent state with mean photon number  $\langle \hat{n} \rangle = 1$  are shown for various values of the smoothing parameter  $\epsilon$ . In the limit when  $\epsilon$  goes to zero  $p(\phi, \epsilon)$  approaches the London phase state distribution (solid line).

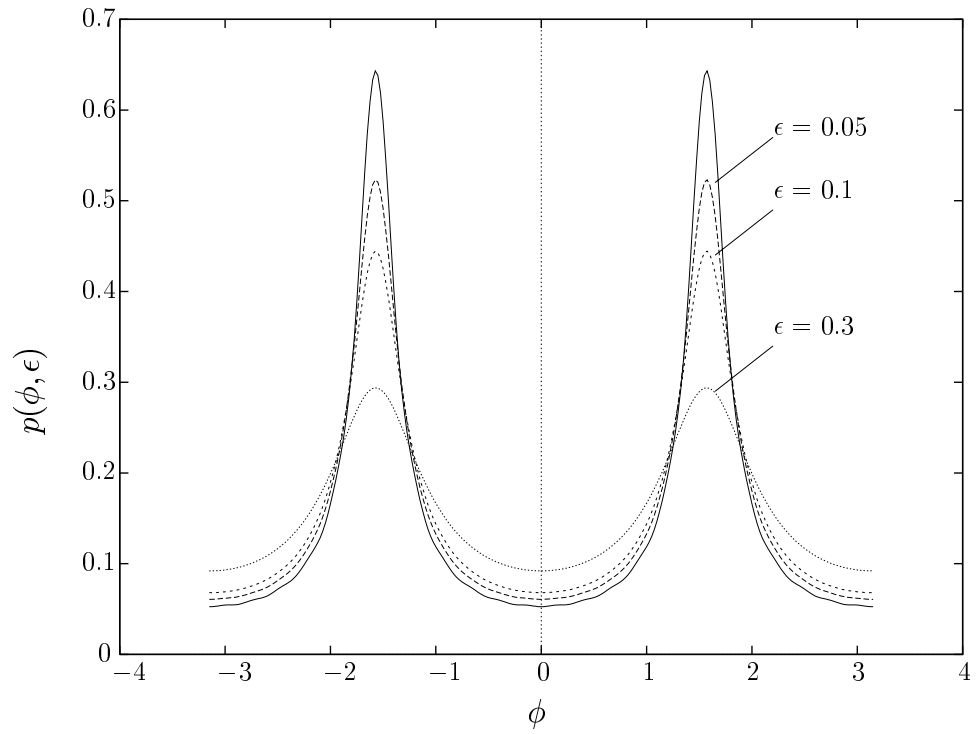


Figure 2: The CPS distributions  $p(\phi, \epsilon)$  for a squeezed vacuum state with mean photon number  $\langle \hat{n} \rangle = 1$  are shown for various values of the smoothing parameter  $\epsilon$ . In the limit when  $\epsilon$  goes to zero  $p(\phi, \epsilon)$  approaches the London phase state distribution (solid line).

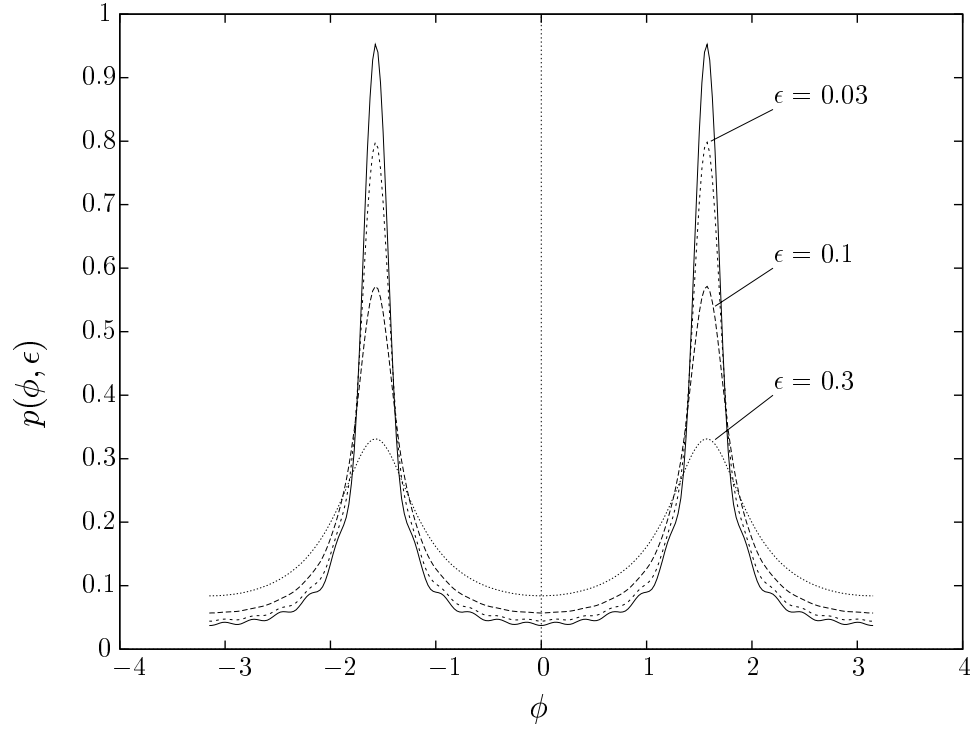


Figure 3: The CPS distributions  $p(\phi, \epsilon)$  for a squeezed vacuum state with mean photon number  $\langle \hat{n} \rangle = 2$  are shown for various values of the smoothing parameter  $\epsilon$ . Note that compared with Fig.2 smaller values of  $\epsilon$  are needed in order to obtain CPS distributions that are comparably close to the London phase distribution (solid line).

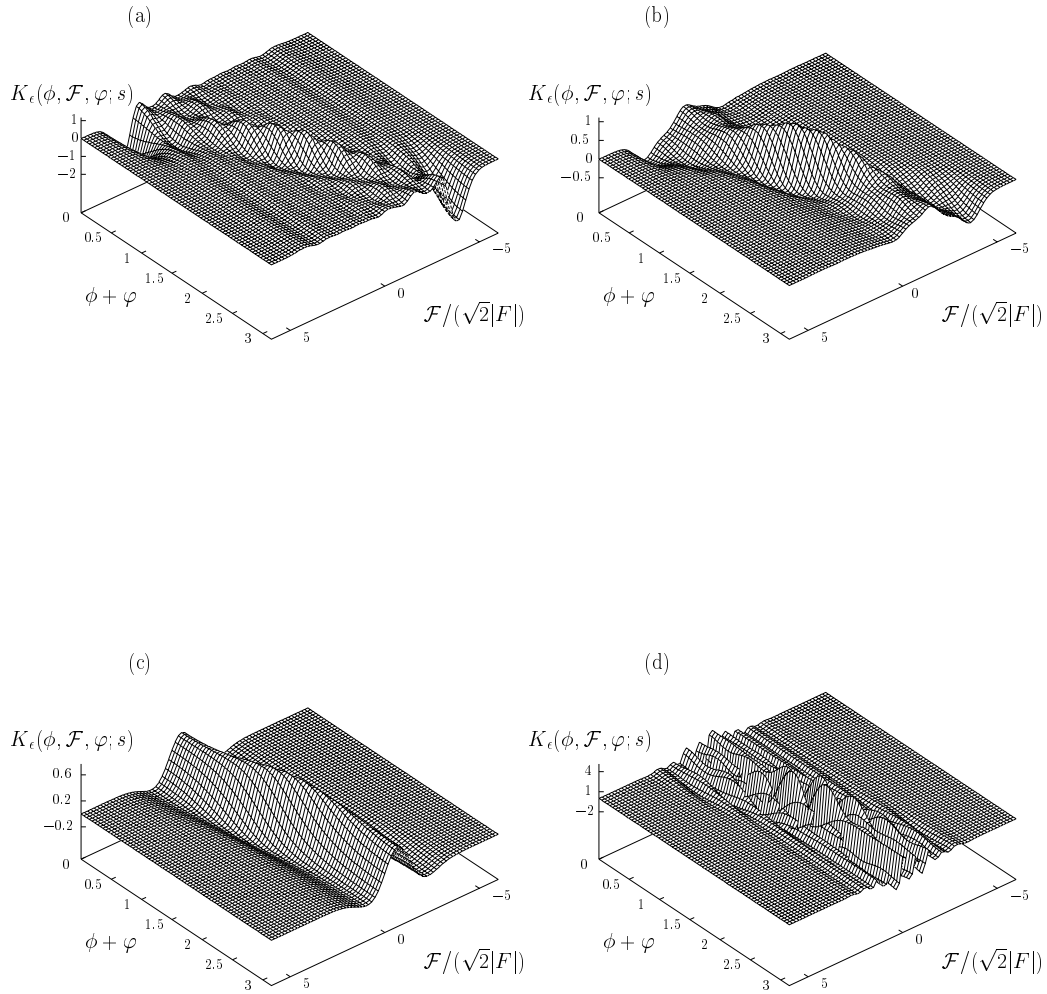


Figure 4:  $K_\epsilon(\phi, \mathcal{F}, \varphi, ; s)$  is shown as a function of  $\mathcal{F}$  and  $\phi + \varphi$  for various values of the smoothing parameter  $\epsilon$  and the detection efficiency  $\eta$ .

- (a)  $\epsilon = 0.1, \eta = 1 (s = 0)$ .
- (b)  $\epsilon = 0.3, \eta = 1 (s = 0)$ .
- (c)  $\epsilon = 0.8, \eta = 1 (s = 0)$ .
- (d)  $\epsilon = 0.1, \eta = 0.8 (s = -0.25)$ .

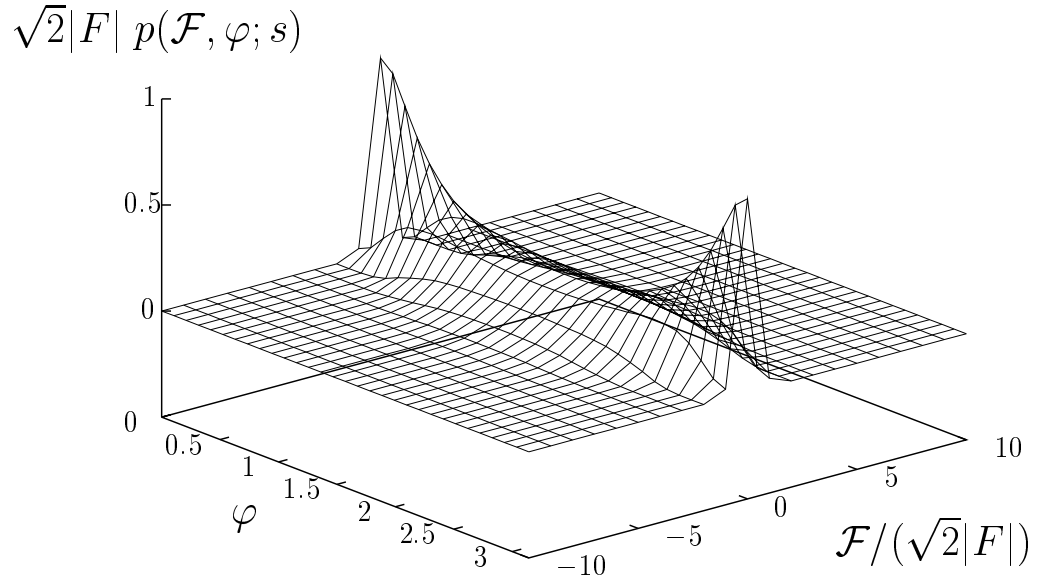


Figure 5: The measured set of field-strength distributions  $p(\mathcal{F}, \varphi; s)$  for a squeezed vacuum state with mean photon number  $\langle \hat{n} \rangle = 1$  is shown. It is obtained from a computer simulation, where perfect detection ( $\eta = 1$ , i.e.,  $s = 0$ ) is considered and  $30 \times 10^4$  events at 30 phase values are assumed to be recorded (the measured, discrete points are linked by straight lines).



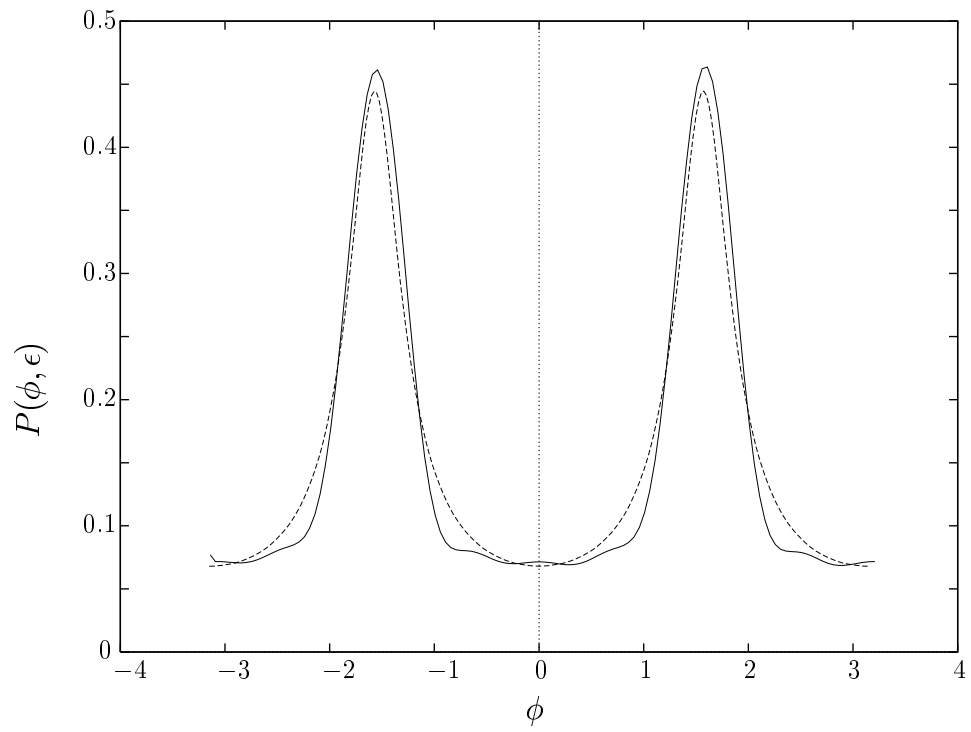


Figure 6: The CPS distribution for  $\epsilon=0.1$  which is directly sampled from the measured field-strength distributions plotted in Fig.5 is shown and compared with the theoretical curve from Fig.2 (dashed line).